1 Introduction

My goal is to convince you that John S. Bell made an egregious error in those of his papers upon which all 'spooky action at a distance', 'quantum teleportation', 'quantum cryptography', etc., experiments are based. These experiments therefore do not prove what they supposedly prove. They are a waste of time and resources.

I will show that Bell's reasoning fails on a simple analogy.

I shall not go into great detail. I will not try to answer the objections of those who introduce assumptions Bell himself never assumed.

It is assumed the reader has a copy of the manuscript version of J. S. Bell, 'Bertlmann's socks and the nature of reality'. This is available at

https://cds.cern.ch/record/142461/

and is freely shareable.

2 Bell assumes his conclusion

Bell presents the general argument starting at p. 11 of the manuscript. Here is where he makes his error.

Let me note, first, that Bell uses a notation resembling that of probability theory, and speaks in terms of probabilities, but does not use any theorems of probability theory in his arguments.[1] Instead he argues in terms of functions representing causal influences, but loosely speaks of them as representing probabilities.[2]

Now let us refer to Bell's equation (11) on p. 13.

To avoid confusing functions representing causation with expressions of probability theory, I will *not* use probability-like notation, but instead functions, such as f, g, and h. Thus, where Bell writes $P(A, B \mid a, b, \lambda)$ to represent the joint outcome at measurement stations A and B as a function of some 'hidden variables' a, b, and λ, I will instead write $h(a, b, \lambda)$. Similarly, instead of $P_1(A \mid a, \lambda)$ and $P_2(B \mid b, \lambda)$, I will write $f(a, \lambda)$ and $g(b, \lambda)$, respectively. The symbol λ represents 'background noises' that have no influence on the outcome, but which, for completeness, are included in the notations.[3]

Bell's equation (11), rewritten in the new notation, looks like this:

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[1] The reader may wish to verify this.

[2] Perhaps this confusion arises from the habit of some physicists to speak of probabilities as physical 'waves'. Probabilities actually are expressions in a mathematical theory built of postulates and theorems.

[3] Bell calls λ 'residual fluctuations'. In any case, λ represents whatever variables there are that do not affect the outcome.

$$h(a, b, \lambda) = h'(f(a, \lambda), g(b, \lambda))$$

That is, the joint outcome at measurement stations A and B is a function h' of the outcomes at the individual stations.[4] To justify writing this, Bell says what I shall paraphrase as follows:

> Note well that we already incorporate a hypothesis of 'local causality' or 'no action at a distance'. For we do not allow $g(b, \lambda)$ to depend on a, nor $f(a, \lambda)$ on b.

From this point onwards, Bell errs: he makes the unjustified assumption that a and b should be treated as *the* independent variables, and not *as functions themselves* of some more fundamental 'hidden variables' that represent *common causes*.[5] In effect, *Bell has assumed his conclusion: that there are no 'hidden variables' capable of representing* common *causes* of the outcomes at measurement stations A and B.

Let us then rewrite equation (11) as it *should* be written. Let us represent by γ the 'hidden variables' representing any common causes. The notation γ may be thought of as a vector of however many individual variables are necessary. Then equation (11) takes this form:

$$h(a(\gamma), b(\gamma), \lambda) = h'(f(a(\gamma), \lambda), g(b(\gamma), \lambda))$$

Note that we still incorporate the hypothesis of 'local causality'! But we no longer assume a and b can be treated as the independent parameters. Instead the variables γ play that role.

None of what Bell concludes from his miswritten equation (11) can be inferred from this corrected form of it, and thus 'Bell inequalities' are experimentally *irrelevant*. Bell wrote his mathematical expressions incorrectly, and as a result was simply *wrong*—and so is anyone else who believes violations of 'Bell inequalities' prove the existence of 'spooky action at a distance', 'quantum teleportation', 'irreducibility of quantum mechanics', etc. Experiments that attempt to (and do) violate 'Bell inequalities' are simply a waste of resources. They prove nothing.

3 A simple thought experiment that defies Bell's reasoning

Let us analyze a thought experiment that has some resemblance to EPR, but which is simpler and easier to visualize. We shall first analyze it correctly, using 'hidden variables'

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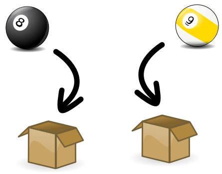
[4] Bell writes h' as multiplication, because he treats the causal joint *outcome* h as if it were itself a joint *probability*, but we shall assume only that h' is *some* function.

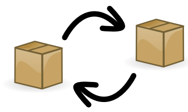
[5] One simply *does not* assume that variables such as a and b are not themselves functions of further parameters! The reader may wish to ponder upon that fact, which pertains *throughout* the mathematical sciences.

that *can* represent common causes. Then we will analyze it *incorrectly*, in the manner of Bell. Here goes:

Put an eight-ball and a nine-ball into indistinguishable boxes, as shown here:



Close the boxes, place them on a table next to each other and mix them up, so no one can tell whether a box contains an eight-ball or a nine-ball, but so one box ends up more to the left and the other more to the right:



Move the two closed boxes to the opposite ends of the table, the box more left going to the left end of the table, and the other box to the right end:

Open the box on the left and remove the ball in it. Let us suppose we observe it to be a nine-ball:

There is no loss of generality in this assumption, because one can simply reverse the roles of the eight-ball and the nine-ball. So let us proceed.

The question we wish to answer is:

> *Will the ball on the right be an eight-ball, or will it be another nine-ball?*

Intuition of course says it will be an eight-ball. But how can we explain this outcome in terms of 'hidden variables'?

Let us define our 'hidden variables'. Let γ_1 represent the state of the ball that, after mixing, ended up on the left. Then $\gamma_1 = 8$ if it is an eight-ball, and $\gamma_1 = 9$ if it is a nine-ball. Similarly, let γ_2 represent the state of the ball that ended up on the right; $\gamma_2 = 8$ if it is an eight-ball, $\gamma_2 = 9$ if it is a nine-ball. Then let

$$\gamma = \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix}$$

be a vector representing the hidden variables. The way the experiment is set up, there are only two possible values of the vector: either

$$\gamma = \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} 8 \\ 9 \end{pmatrix}$$

or

$$\gamma = \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} 9 \\ 8 \end{pmatrix}$$

We have assumed the latter, without loss of generality.

Let us further define functions a and b, as follows: the function a, applied to γ, selects the first element of the vector:

$$a(\gamma) = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} = \gamma_1$$

The function b, on the other hand, selects the second vector element:

$$b(\gamma) = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} = \gamma_2$$

In other words, a selects the 'hidden variable' that is relevant to outcome on the left side of the experiment, whereas b selects the 'hidden variable' that is relevant to outcome on the right side. This choice gives $a(\gamma)$ and $b(\gamma)$ the same role as the 'hidden variables' in Bell's equation (11), but now they are *functions* of the *actual* independent variables, γ.

Substituting $\gamma_1 = 9$ and $\gamma_2 = 8$, we thus have $a(\gamma) = 9$ and $b(\gamma) = 8$. Notice that $b(\gamma)$ represents an eight-ball. If the 'measurement' function g simply filters out all the noise λ and returns $b(\gamma)$, then our 'hidden variables' theory predicts that the ball in the box on the right will be an eight-ball. This result accords with intuition. It also accords with actual outcomes, if one carries out the experiment for real:

In other words, the prediction is *correct*. The theory does what a 'hidden variables' theory is supposed to do: it explains the chain of causation, without 'action at a distance'.

From an analysis similar to Bell's, however, one gets a *different* prediction. Astonishingly, according to a Bell-like analysis, the ball on the right will be an eight-ball half the time, but half the time it will be *a second nine-ball!*[6]

For, if we apply Bell's methods, we no longer have a and b as functions of 'deeper hidden variables' γ, but rather as independent variables in their own right. One must agree that, at the stage where the two boxes have been separated to their respective 'measurement stations' at the ends of the table but the boxes are still unopened, there is probability one-half that $a = 8$ and probability one-half that $a = 9$. Similarly, there is probability one-half that $b = 8$ and probability one-half that $b = 9$. But we do not allow b to affect the outcome on the left, nor a to affect the outcome on the right. Thus, even though we have gone ahead and *observed* that $a = 9$, we still have (according to a Bell-like analysis) probability one-half that $b = 8$ and probability one-half that $b = 9$. Half the time we should observe an eight-ball mysteriously replaced by a nine-ball!—

Of course this prediction is ridiculous, but it comes about by applying Bell's faulty logic. We can dismiss Bell's analysis as fundamentally incorrect.[7]

[6] To carry out Bell-like reasoning further: experimentally, only an eight-ball ever is observed, so we must conclude 'local reality' is a false assumption. In other words, observing a nine-ball on the left *must be causing* the ball on the right to become an eight-ball, by some kind of 'instantaneous action at a distance'.

[7] Thus Bell did *not* discover a contradiction between 'local reality' and quantum mechanics. Quantum physicists, once they realize there are no impediments but orthodoxy and faulty logic, may proceed to develop the long overdue 'hidden variable' theories.