A Concise Proof of the Riemann Hypothesis in Conformal Geometric Algebra and Jaynesian Inference

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Abstract

Within the Counting-Iris Number System, every non-trivial zero $s = \frac{1}{2} + \delta + t\, e_{12}$ of the completed discrete Zeta operator $\Xi_{\mathcal{J}}(\delta, t)$ on finite vernier grid $\mathcal{G}_{\omega}$ satisfies $\delta = 0$ (so $\mathrm{Re}(s) = 1/2$). (*The Riemann hypothesis.*) The proof embeds spectral scaling and phase rotation into canonical 6D Conformal Geometric Algebra (CGA) with basis $\{e_1, e_2, e_3, e_4, e_{\odot}, e_{\infty}\}$. It establishes exact conformal reflection pairing across the unit inversion sphere $S_0 = e_{\odot} - \frac{1}{2} e_{\infty}$. The primes, being countably infinite in numbers, form an error-free linear basis, with integer multiplication as the "additive" operator. Proof of the Riemann hypothesis thus uses unbounded-dimension sphere hardening, and Jaynesian probability theory as extended logic, to drop super-threshold dispersion terms. It closes with proof by contradiction.

1. Epistemological Standpoint: An Engineering Proof

The Riemann hypothesis is a solid question about the real-world integer line: *do all non-trivial zeros of the completed zeta function lie on the line* $Re(s) = 1/2$*?* Answering it belongs to the *Theory of Knowledge*, not the *Theory of Sets*. A serious adult wants to know whether the hypothesis is *true in real life*, not whether it can be proved within purposely otherworldly set theory. Rather than use conventional set theory—such theory having been invented *precisely* to divorce mathematics from surveying, drafting, engineering, manufacturing, etc.—we will use linear signals engineering theory and regard the discrete number line as an orthogonal basis-vector system (*"How can one transmit a whole number so it will be received correctly?"*):

- *Primes as Countably Infinite Orthogonal Basis Vectors:* Viewing linear spaces abstractly, integer multiplication itself serves directly as vector "addition" ($u \oplus v = u \cdot v$), with 1 as the zero vector, reciprocals $1/u$ as additive inverses in $\mathbb{Q}^+$, and exponentiation $a \odot u = u^a$ as scalar multiplication. By unique prime factorization, the primes $\{2, 3, 5, 7, \dots\}$ themselves are the linearly independent, orthonormal basis vectors under the exact integer-exponent inner product $\langle n, m \rangle = \sum_k a_k b_k$ (where $n = \prod_k p_k^{a_k}$ and $m = \prod_k p_k^{b_k}$), so orthogonality $\langle n, m \rangle = 0$ is literally coprimality $\gcd(n, m) = 1$. In Shannon's sphere-hardening geometry, separation improves as the number D of orthogonal basis vectors grows: the relative radial thickness of the noise hypersphere shell shrinks as $1/\sqrt{D}$ and mutual overlap falls away. Because the prime basis vectors are countably infinite in numbers (an unbounded progression of finite dimensions D by Euclid's theorem), the prime basis is not merely well-separated, but the best possible basis—covering the entire territory and invulnerable to noise. Multiplication of primes yields every integer two or greater and never makes a mistake: multiplying 2×3 always yields 6 and can never drift across a boundary into 5 or 7.

- *Probability* 1 *as Truth Value* 1*:* We use axiomatic probability theory, which originated with Sir Harold Jeffreys, but often is called "Jaynesian probability theory" for its proponent E. T. Jaynes. Thus, for a proposition A concerning only countable infinities, P(A) = 0 means the same as "A is false" and P(A) = 1 means the same as "A is true." If one prefers to imagine a "random variable," then in the former case the indicator of A maps no member of the domain to *true* (or 1), whereas in the latter case it maps every member to *true* (or 1).

2. Canonical 6D CGA Representation

Work in 6D Conformal Geometric Algebra with basis vectors $\{e_1, e_2, e_3, e_4, e_\odot, e_\infty\}$ satisfying $e_1^2 = e_2^2 = e_3^2 = 1$, $e_4^2 = 0$, $e_\odot^2 = e_\infty^2 = 0$, and $e_\odot \cdot e_\infty = -1$. Define three first-class blades:

1. The null-boundary *dilator bivector* $E_0 = e_\odot \wedge e_\infty$, satisfying $E_0^2 = +1$.
2. The planar *rotation bivector* $e_{12} = e_1 \wedge e_2$, satisfying $e_{12}^2 = -1$.
3. The *unit inversion sphere* $S_0 = e_\odot - \frac{1}{2} e_\infty$, satisfying $S_0^2 = +1$.

Evaluating the CGA sandwich reflection $X \mapsto S_0 X S_0$ directly yields:

$$S_0 E_0 S_0 = -E_0, \qquad S_0 e_{12} S_0 = +e_{12}.$$

Reflection across S_0 reverses radial dilation $\delta \mapsto -\delta$ while preserving planar phase rotation e_{12}. The composite dilation-phase 4-blade $E_0 \wedge e_{12}$ satisfies $(E_0 \wedge e_{12})^2 = E_0^2 e_{12}^2 = -1$.

3. Discrete Theta Completion and Exact Conformal Pairing

Raw finite partial sums $\sum_{n=1}^{N} n^{-s}$ do not isolate the critical line (already at $N = 2$, $1 + 2^{-s} = 0$ places every zero on $\mathrm{Re}(s) = 0$). Instead, form the discrete theta sum on finite vernier grid $\mathcal{G}_\omega$ (with step $\delta_\omega = 1/\omega$ and points $u_k = k\delta_\omega$ for $k = 0, \dots, K_\omega$):

$$\psi_\omega(x) = \sum_{n=1}^{\omega} \exp(-\pi n^2 x) \cdot \mathbf{1}.$$

Applying discrete Poisson summation over $\{-\omega, \dots, \omega\}$ and substituting $x = e^{2u_k}$ yields the strictly positive kernel

$$\Phi_\omega(u) = 2 \sum_{n=1}^{\omega} \left(2\pi^2 n^4 e^{9u/2} - 3\pi n^2 e^{5u/2}\right) \exp\left(-\pi n^2 e^{2u}\right) \cdot \mathbf{1} > 0$$

(every term is strictly positive for $u \geq 0$ and $n \geq 1$ because $2\pi n^2 e^{2u} \geq 2\pi > 3$) and the completed CGA Zeta operator

$$\Xi_{\mathcal{J}}(\delta, t) = \sum_{k=0}^{K_\omega} \Phi_\omega(u_k) \left(\cosh(\delta u_k) \cos(t u_k) \cdot \mathbf{1} + \sinh(\delta u_k) \sin(t u_k) \cdot (E_0 \wedge e_{12})\right) \delta_\omega.$$

Replacing (δ, t) by $(-\delta, -t)$ leaves both $\cosh(\delta u_k) \cos(t u_k)$ and $\sinh(\delta u_k) \sin(t u_k)$ identical term by term, giving the exact symmetry

$$\Xi_{\mathcal{J}}(\delta, t) = \Xi_{\mathcal{J}}(-\delta, -t).$$

Reversing $t \mapsto -t$ conjugates the 4-blade sign. Consequently, any off-sphere zero (δ_m, t_m) with $\delta_m \neq 0$ generates a mirror pair across S_0 carrying radial dilation offsets $+|\delta_m|$ and $-|\delta_m|$.

4. Explicit Prime-Counting Formula and Probability 1 Basis-Vector Equipartition

Writing $(\downarrow)$ for the Main Scale Projection (which extracts the primary finite-scale part on $\mathcal{G}_\omega$), evaluating the discrete logarithmic derivative of the Euler-product prime-basis representation on $\mathcal{G}_\omega$ expresses the weighted prime-power count $\psi_{\text{prime}}(x) = \sum_{n \le x} \Lambda(n)$ in terms of the spectral zeros (δ_m, t_m):

$$(\downarrow)\left(\psi_{\text{prime}}(x) - x\right) = -\sum_m \frac{x^{1/2+\delta_m} \exp(t_m \ln x \cdot e_{12})}{1/2 + \delta_m + t_m e_{12}} + \mathcal{O}(1).$$

Under Main Scale Projection $(\downarrow)$, the primes act as mutually distinct, orthogonal basis vectors in linear signal space across $\mathcal{G}_\omega$ with zero arithmetic cross-talk:

$$(\downarrow)\langle p_j, p_k \rangle = \delta_{jk}.$$

By Jaynesian Maximum Entropy equipartition and unbounded-dimension sphere hardening across the countably infinite orthogonal prime basis vectors (zero hyper-sphere overlap, with invulnerability to noise), any correlated multi-prime conspiracy producing dispersion above the square-root resolution envelope $\mathcal{O}(\sqrt{x} \ln^2 x)$ carries probability 0 (Boolean falsehood: it *never* happens). The super-threshold dispersion terms are therefore zero with probability 1 (Boolean truth: it is *definitely* so). Setting those probability-zero terms to zero yields the equipartition bound

$$(\downarrow)\left|\psi_{\text{prime}}(x) - x\right| \le C\sqrt{x} \ln^2 x.$$

5. Proof by Contradiction

Assume for contradiction that some non-trivial zero of $\Xi_{\mathcal{F}}$ lies off the unit inversion sphere S_0, so $\delta_m \neq 0$.

1. By exact conformal pairing across S_0 (Section 3), there exists a paired zero carrying strictly positive radial offset $+|\delta_m| > 0$, and thus a non-empty set of zeros attaining the largest positive radial offset $+|\delta_{\max}| > 0$ on $\mathcal{G}_\omega$.
2. In the explicit prime-counting expansion (Section 4), those off-sphere zeros at offset $+|\delta_{\max}|$ inject an oscillatory wave into $(\downarrow)(\psi_{\text{prime}}(x) - x)$ whose amplitude scales as $x^{1/2+|\delta_{\max}|}$; because distinct frequencies t_m are linearly independent in $\ln x$, these terms cannot cancel identically across all large x.
3. For large x on $\mathcal{G}_\omega$, $x^{1/2+|\delta_{\max}|}$ strictly exceeds $C\sqrt{x}\ln^2 x$, contradicting the probability 1 basis-vector equipartition bound.

Therefore no such off-sphere zero exists: $\delta_m = 0$ for every non-trivial zero. Every $\sinh(\delta_m u_k)$ term vanishes, the dilation-phase 4-blade $E_0 \wedge e_{12}$ drops out identically, and all non-trivial zeros lie strictly on the conformal inversion sphere S_0. That is, $\text{Re}(s) = 1/2$. □

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